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Mathematics Department

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www.bu.edu.eg/staff/mohamedeed3-course

Student → **Program(courses)** → **Engineer**

1. **Knowledge and Understanding**
2. **Intellectual Skills**
3. **Professional and Practical Skills**
4. **General Skills**

Course Aims

- **To provide the students essential information and fundamentals of Differential and Integral Calculus and their applications in engineering.**
- **To apply mathematical techniques for modeling, solving and analyzing real problems.**

Intended Learning Outcome

a- Knowledge and Understanding

1. Identify theories and fundamentals of mathematics.
2. Describe principles of mathematics for treating real problems.

b- Intellectual Skills

1. **Analyze mathematical problems and categorize them.**
2. **Solve practical problems using mathematical methods.**
3. **Make mathematical models to real problems in the light of available data and information.**

c- Professional and Practical Skills

1. Apply mathematical logic and techniques for solving real life problems
2. Prepare professional reports via mathematical logic.

d- General Skills

1. Work in a group and lead a team
2. Manage time and conduct self learning and continuous education
3. Use technology for obtaining information and knowledge
4. Communication skills

Contents

Student Assessment

Procedures

- Exercises
- Discussions and presentations
- Quizzes
- Exams

Weighting of assessments

- **Final-semester examination 60 Marks**
- **Mid-semester examination 20 Marks**
- **Quizzes 10 Marks**
- **Class activities 10 Marks**

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- **Total 100 Marks**

List of References

1- Course Notes

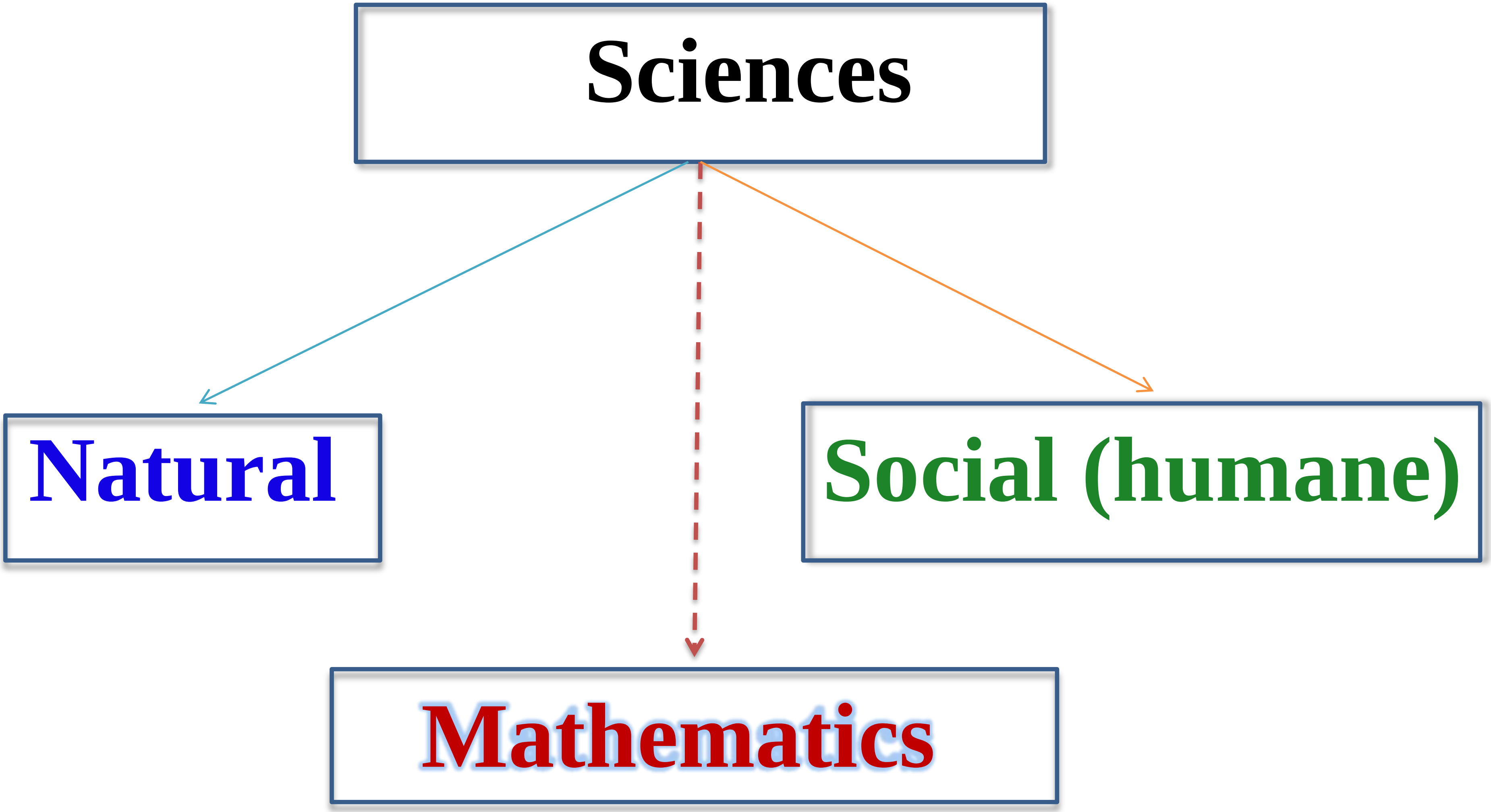
- **''Lectures In Mathematics '', Differential and Integral Calculus, Mohamed H. Eid, Benha Univeristy, 2012.**

2- Text Books

- **''Calculus'', 6th Edition, James Stewart, Thomson Brooks / Cole, U.S.A, 2008.**

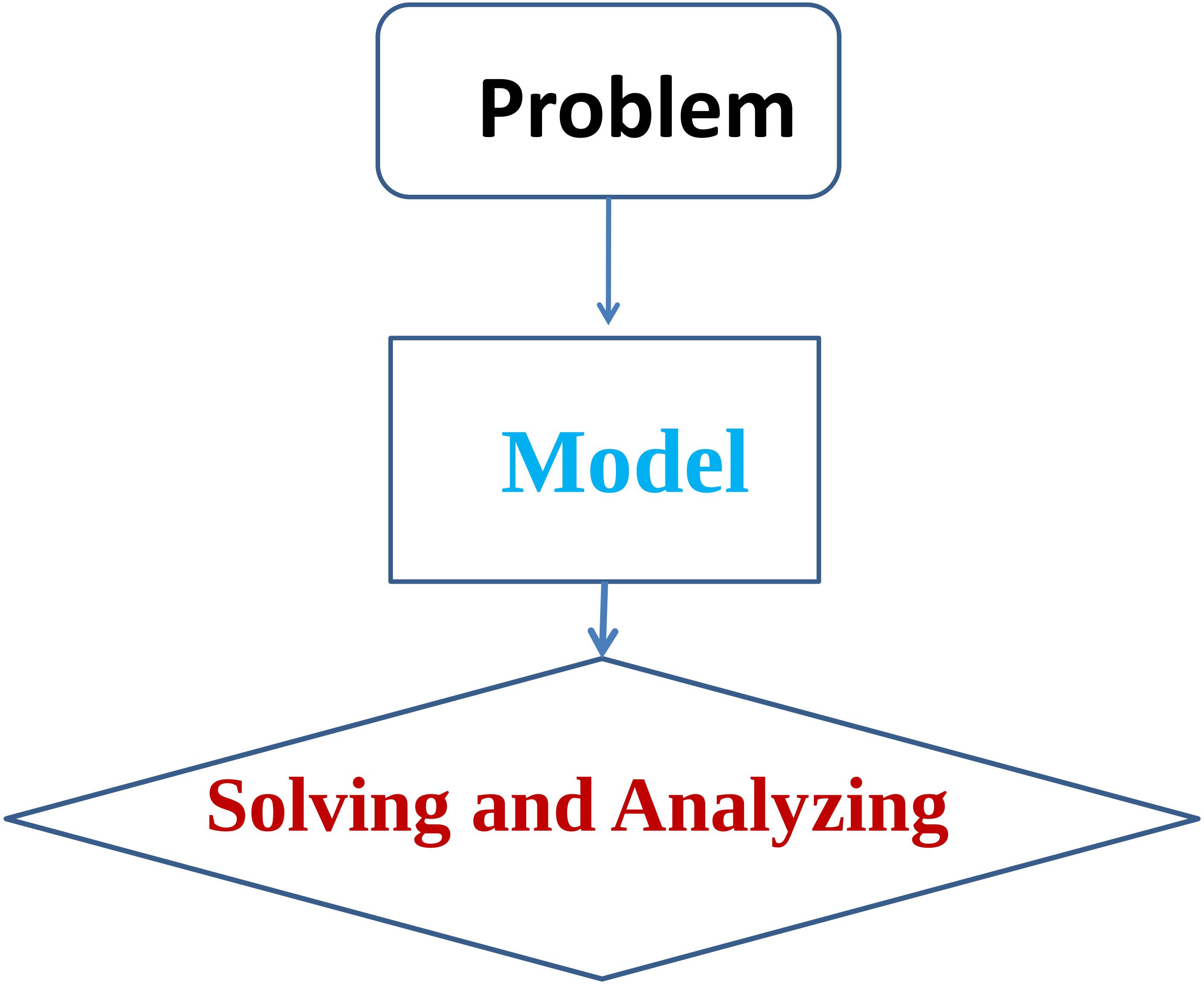
3- Recommended Books

- **''Advanced Calculus With Applications In Statistics'', 2nd Edition, A.I. Khuri, John Wiley & Sons, Inc., New Jersey, 2003.**

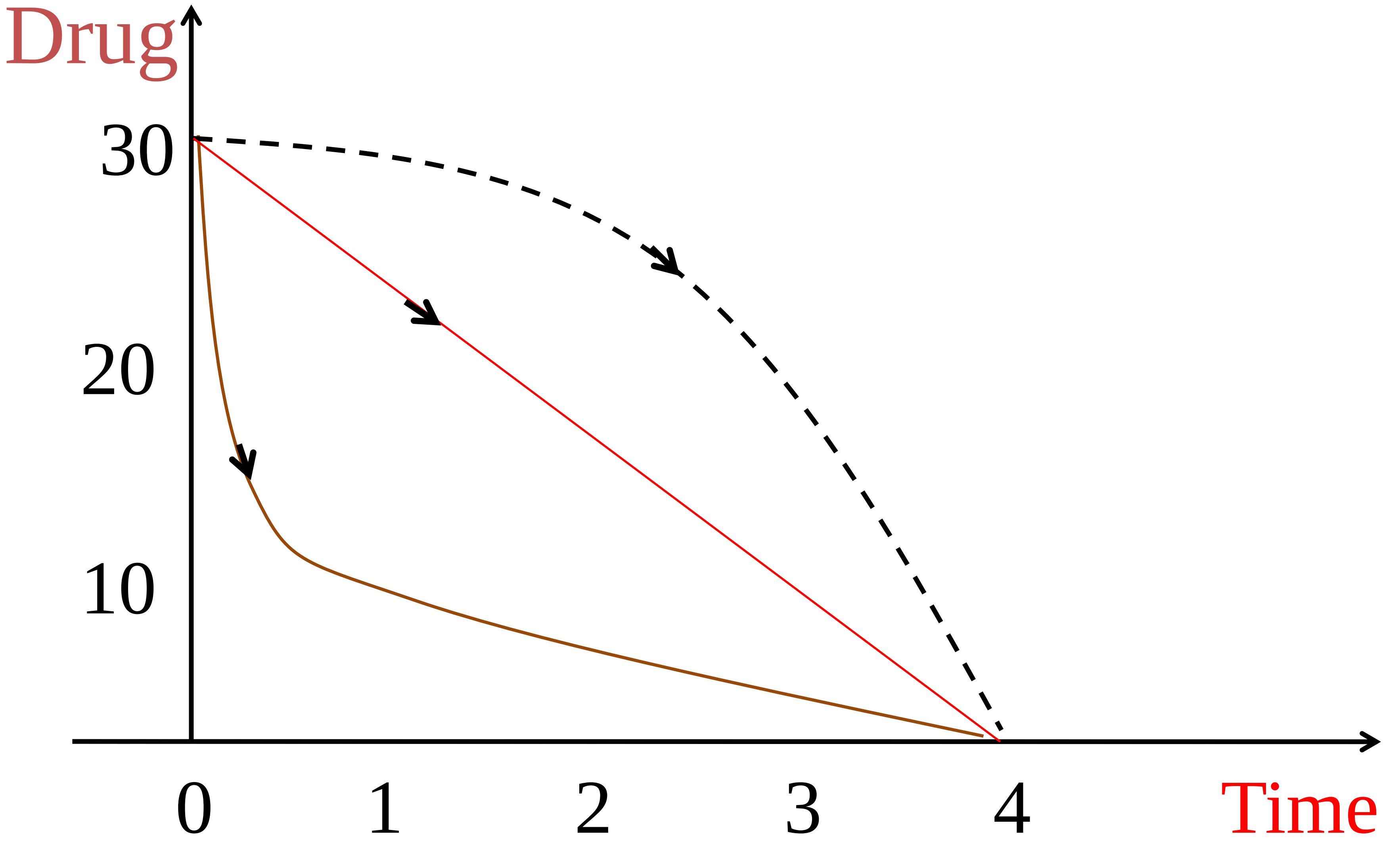


Mathematics is the science of modeling and treatment problems and phenomena via explicit criteria

Mathematics



Graph - Rate of Change of Concentration



Rate of Change

Example: An amount of sugar (100 gm) in solution is decomposed in a chemical reaction into other substance through the presence of acids, and the rate at which the reaction takes place is proportional to the mass of sugar still unchanged.

Write the mathematical model.

Find the time at which all amount is decomposed

تتحلل كمية من السكر (100 جم) في محلول في تفاعل كيميائي إلى مادة أخرى من خلال وجود الأحماض،

و معدل التغير يتناسب مع كتلة السكر المتبقية.

The original amount of sugar is 100 gm.

Assume that **x** is the amount of sugar converted at time t.

Then $100 - x$ is the amount still unchanged

Then $\frac{dx}{dt} = k(100 - x)$, K is constant, k =1

Then $\frac{dx}{x-100} = -dt$

Then $\ln(x-100) = -t + c$

Then $x - 100 = e^{-t+c} = C.e^{-t}$

The decomposition starts when $t = x = 0$

Then $0 - 100 = C.e^{0} = C$

Then $x = 100 - 100e^{-t} = 100(1 - e^{-t})$

is the mathematical relation.

(Increasing relation)

From $x(t) = 100(1 - e^{-t})$

t / minute	**x / gm**
1	63.2
2	86.5
4	98.2
5	99.99

All amount of sugar is converted when

$x = 100$ gm, t approaches infinity

Example

Chemical A is being converted into chemical B at reaction rate -0.5 per second. The initial concentration of A is 10 moles/liter.

Determine the concentration $C(t)$ as a function of the time t.

Find the time at which the concentration C is 5 moles/liter.

The mathematical relation is $\frac{dC}{dt} = -\frac{1}{2}C$

Then $\ln C = -0.5t + k$

Then $C = e^{-0.5t+k} = m.e^{-0.5t}$

At $t = 0$, $C(0) = 10 = m.e^{0}$. Then $m = 10$

Then $C(t) = 10e^{-0.5t}$

is the mathematical relation.

(Decreasing relation)

From $C(t) = 10e^{-0.5t}$

t / second	C moles / liter
0	10
1	6.065
2	3.679

When C = 5, then $5 = 10e^{-0.5t}$

Then t = 1.4 seconds

Example: Mixing Solution

A tank contains 100 liters a brine solution containing 20 kg of salt. At time $t = 0$, fresh water is poured into the tank at rate 4 liters per minute while the well mixture leaves the tank at the same rate.

Determine the amount of salt in the tank at any time t.

خزان يحتوي على 100 لتر محلول ملحي يحتوي على 20 كجم من الملح. في الزمن $t = 0$، يتم سكب المياه العذبة في الخزان بمعدل 4 لتر في الدقيقة بينما الخليط المخفف يخرج بنفس المعدل.

If S is the amount of salt in kg at any time

The concentration in kg in liter is S/100

Then $\dfrac{dS}{dt} = -4\dfrac{S}{100} = -0.04\,S$

Then $S(t) = e^{-0.04t+k} = m.e^{-0.04t}$

At $t = 0$, $S(0) = 20 = m.e^{0}$. Then $m = 20$

Then $S(t) = 20e^{-0.04t}$

is the mathematical relation.

(Decreasing relation)

From $S(t) = 20e^{-0.04t}$

t / minute	**S / Kg**
0	20
1	19.22
2	18.46
10	13.4

The amount of salt in solution is 0 when t approaches infinity

Example

A metal bar at a temperature of 100^0 F is placed in a room at a constant temp. 0^0 F. After 20 minutes the temp. of the bar is 50^0

Find the time at which the temp. of the bar is 25^0

Find the temp. of the bar after 10 minutes.

Assume that u is the temp. of the bar at time t.

From Newton's law of cooling

$$\frac{du}{dt} = -k(\text{temp.of bar} - \text{temp.of its surrounding})$$

$$= -k(u - 0)$$

Then $\frac{du}{u} = -kdt$ Then $\ln u = -kt + c$

Then $u = e^{-kt+c} = e^{c}.e^{-kt} = C.e^{-kt}$

Since $u(0) = u(\text{time} = 0) = 100^0$

$u(20) = u(\text{time} = 20) = 50^0$

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Then $u = e^{-kt+c} = e^{c}.e^{-kt} = C.e^{-kt}$

Since $u(0) = u(\text{time} = 0) = 100^0$

$u(20) = u(\text{time} = 20) = 50^0$

Then $100 = C.e^{0} = C$

$$50 = 100e^{-20k}, \text{ then } k = 0.035$$

The mathematical relation is:

$$u(t) = 100e^{-0.035t}$$

When the temp. of the bar is 25^{0}

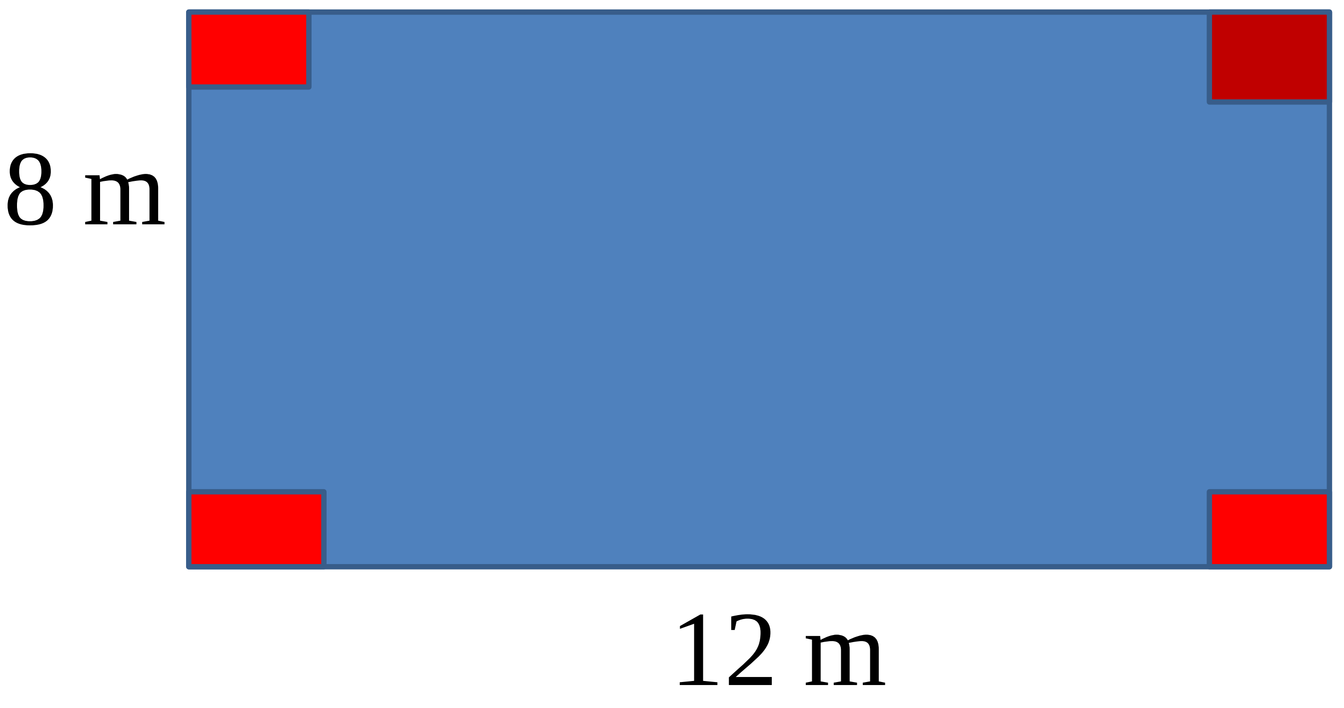
Then $25 = 100e^{-0.035t}$, then $t = 39.6$ min

After 10 minutes, the temp. of the bar is:

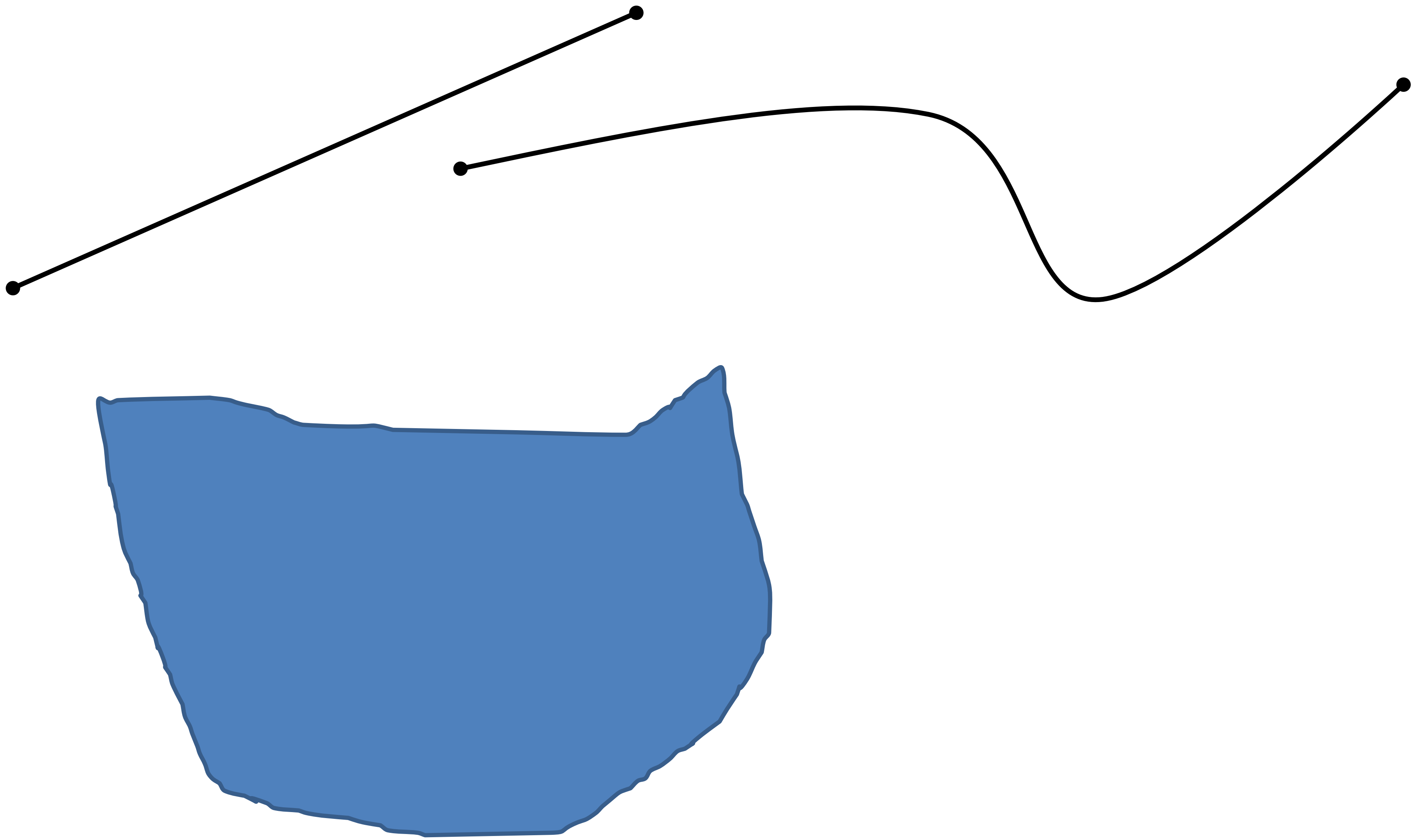
$$u(10) = 100e^{-0.035(10)} = 70.5^{0}\ F$$

Optimization Problem

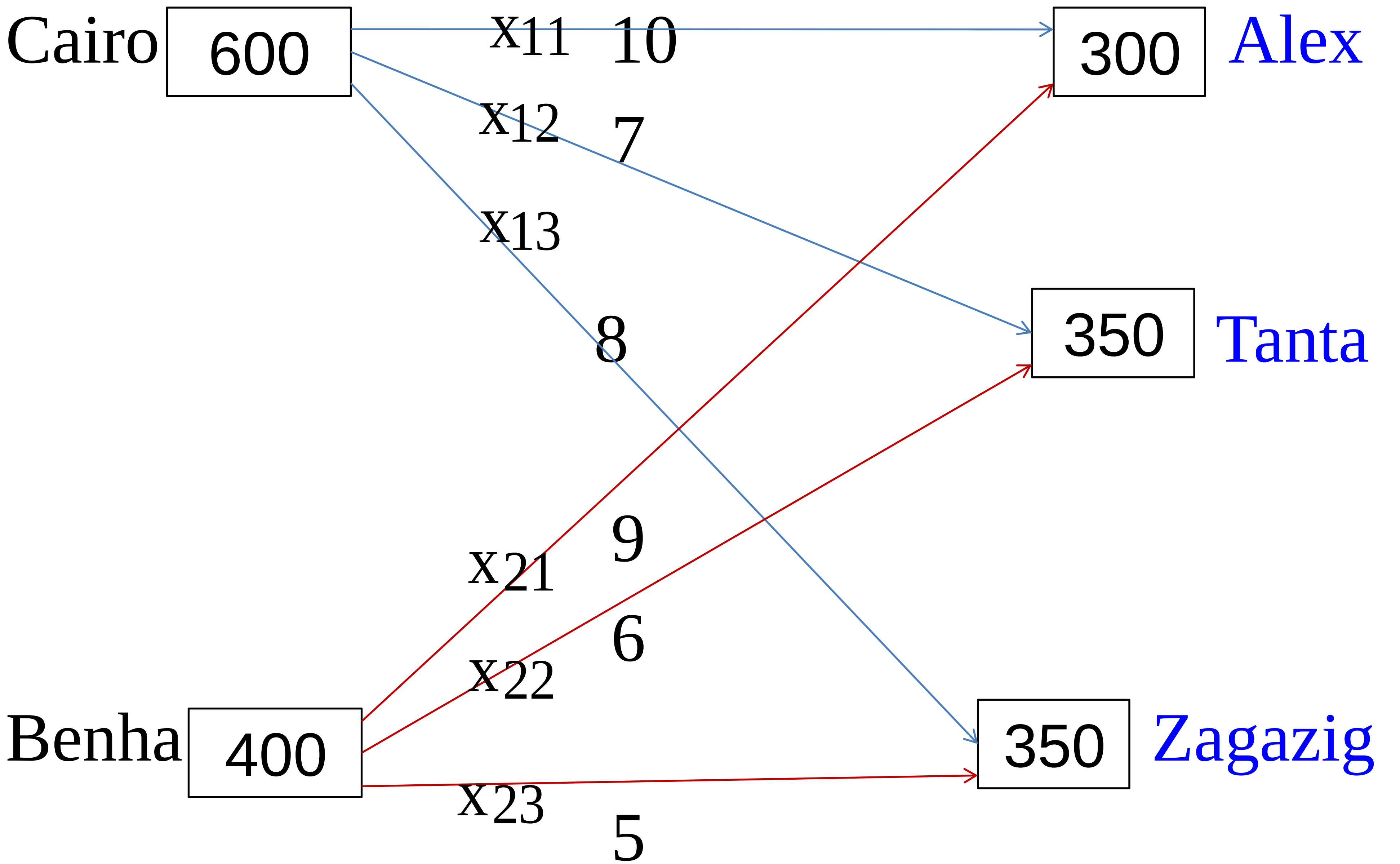
Design a Box



Application of Integral



Optimization Problem



Mathematical Model

$$\text{Minimize} \quad f = 10x_{11} + 7x_{12} + 8x_{13} + 9x_{21} + 6x_{22} + 5x_{23}$$

$$\text{s.t} \quad x_{11} + x_{12} + x_{13} = 600$$

$$x_{21} + x_{22} + x_{23} = 400$$

$$x_{11} + x_{21} = 300$$

$$x_{12} + x_{22} = 350$$

$$x_{13} + x_{23} = 350$$

$$x_{11},\ x_{12},\ x_{13},\ x_{21},\ x_{22},\ x_{23} \geq 0$$

Thank You